Dual Dig Level I (2011)

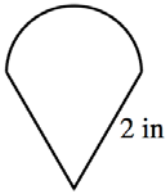
1. Pat is thinking of a positive integer. The sum of its square and its (positive) square root is 84. What integer is Pat thinking of?

2. A quadrilateral *ABCD* has four interior angles measured in degrees. Angle *B* measures 20° more than Angle *A*. Angle *C* measures 20° more than Angle *B*. Angle *D* measures 20° more than Angle *C*. What is the measure of Angle *A*?

3. You have three separate boxes and inside each box are two separate smaller boxes, and inside each of these smaller boxes are four even smaller boxes. What is the total number of boxes?

4. Simplify: $(x+1)+(x-2)+(x+3)+(x-4)+...+(x+99)+(x-100)$

5. A first-grade student daydreams about an ice cream cone and (carefully) draws the figure below (not to scale). The figure consists of a semicircle and two sides of an equilateral triangle of side length two inches, with the missing side of the triangle serving as the diameter of the semicircle. What is the perimeter of the figure?



6. Find all real solutions of the equation: $\frac{x}{2}+\frac{2}{x}=\frac{x}{3}+\frac{3}{x}$

7. For the positive integer *n*, let <*n*> denote the sum of all positive divisors of *n* with the exception of *n* itself. For example, <4> = 1 + 2 = 3, and <12> = 1 + 2 + 3 + 4 + 6 = 16. What is <<< 6 >>> ?

8. Find the <u>inverse</u> of the function f, where: $f(x)=\frac{2x-5}{x+6}$

9. Suppose $f(x,y)$ is some function such that: $f(x,y)=3x^2-2y^2+xy$.
   Find: $f\left(f(1,2),\ f(2,1)\right)$

10. There exists a triangle that has the points $(0,0),(1,0)$, and $\left(1,\sqrt{3}\right)$ as its vertices. What is the mean of the distances from each vertex to the midpoint of the hypotenuse?

11. Four roommates are all late for their Calculus Final Exam. They rush into the classroom and tell the teacher, "We came to school in the same car and had a flat tire, and by the time we changed it, we'd missed the Final." The teacher put each student in a different corner of the room, gave each one a single piece of paper, and asked them to indicate which tire went flat. The teacher picked up the papers after 15 seconds. What is the probability that they all selected the same tire, assuming their selections were random? (Disregard the 'spare tire.')

12. For some unknown positive base *x*, the following are true: $\log_x 3 = A$, and $\log_x 2 = B$. Express $\log_x(0.75x^3)$ in terms of *A* and/or *B*.

13. Simplify the fraction completely: $\dfrac{\left(3^{2011}\right)^2 - \left(3^{2009}\right)^2}{\left(3^{2010}\right)^2 - \left(3^{2008}\right)^2}$

14. Kenny uses the following procedure to write down a sequence of numbers. First he chooses the first term to be 6. To generate each succeeding term, he flips a fair coin. If it comes up heads, he doubles the previous term and subtracts 1. If the coin comes up tails, he takes half of the previous term and subtracts 1. What is the probability that the fourth term in Kenny's sequence is an integer?

15. If $\dfrac{1}{x^3} - \dfrac{1}{x^2} - \dfrac{1}{x} - 1 = 0$, then what is the value of $x^3 + x^2 + x + 1$?

16. Circle #1 and Circle #2 are concentric circles ("concentric" means "sharing the same center"). Circle #1 has radius *x*. Circle #2 is a larger circle drawn outside of Circle #1 so that the area of the ring (the region outside of Circle #1, but inside of Circle #2) has exactly the same area as Circle #1. Find a simplified expression for the radius of Circle #2.

17. In the expansion of $\left(x^5 + x\right)^{100}$ written in descending powers of *x*, what is the coefficient of x^{496}?

18. Convert 523 (base 7) to a base 5 numeral.

19. Pat and Riley work for the same company. Pat says, "$19/40$ of my co-workers are female." Riley says, "$12/25$ of my co-workers are female." (a) How many total workers are there (including Pat and Riley). (b) Is Pat female? (Yes or No). Is Riley female? (Yes or No).

20. Suppose you write out all the integers from 1 to 1000 inclusive. How many times would you write the digit '1'?